SIMULATING THE REFRIGERATION OF BATCH DAIRY PRODUCTS IN A MULTIZONE COLD SUPPLY SYSTEM

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Abstract: Methods that simulate the fast refrigeration of foods on the basis of a model of adjustable heat sink according to the principle of programmed freezing are considered. In this case, a fast freezer is seen as a system of modules, each of which can independently ensure the necessary heat-sink conditions for the fast refrigeration process. The focus is made on the analysis of physicochemical processes that form the water crystallization front at the first freezing stage, taking into account the thermophysical specifics of organizing a multizone combined system of refrigeration supply. Test-bench studies were conducted to obtain the main regularities of fast freezing of single-piece packaged dairy products by the nitrogen + air combined method in a wide range of heat-exchange conditions. The fast freezer has two freezing zones with various temperatures, allowing an efficient distribution of energy costs and creating the optimal conditions for freezing and for the continuity of the technological cycle. A mathematical model has been developed on the basis of experimental data analysis to determine the main technological parameter, the duration of food refrigeration in a nitrogen + air combined two-zone fast freezer with adjustable heat sink. The integral characteristics of the mathematical model have been determined. The model's adequacy to the real freezing process has been proved.

Keywords: combined method, refrigeration, dairy products, fast freezer, nitrogen, temperature, zone, duration, calculations

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INTRODUCTION

The simulation of refrigeration processes used for various foodstuffs and raw materials with precise goal setting and results obtained has been conducted by many authors, such as I.G. Alyamovskii, A.M. Brazhnikov, K.P. Venger, D.G. Ryutov, G.B. Chizhov, I.G. Chumak, and A.P. Sheffer. These studies were based on the theories of Planck, Stefan, Lamé, and Clapeyron, and the solution of the problem was reduced to determining the duration of food freezing to a preset volume-averaged or final temperature in the middle of the body in a criterion, dimensionless, or classical form.

The simplest dependence for determining the duration of the refrigeration process was developed by Planck. This solution is considered classical, being notable for its simplicity and ease of use. It is built on the following assumptions:

– a homogeneous moisture-containing body is cooled to the cryoscopic temperature before refrigeration;

– ice formation occurs without overcooling and isothermally at the cryoscopic temperature, and the thermophysical properties of the frozen part of the body's total volume do not depend on temperature, the thermal capacity of the frozen part being equal to zero, and

– refrigeration occurs by removing heat from the body surface, the heat transfer coefficient and the temperature of the heat-sinking media being constant.

The first analytical solution to the problem of the duration of freezing a flat plate from the initial temperature, which is higher than the cryoscopic one, to the final temperature of the middle, which is below the cryoscopic one, was obtained by D.G. Ryutov and was widely recognized in refrigeration technology [1].

In order to take into account the duration of the plate's temperature decline after the convergence of phase boundaries, a linear temperature change is admissible along the thickness of the frozen layer. Further heat exchange in the plate is analyzed on the basis of the regularities of simple cooling. The time during which the temperature of the plate center decreases from the cryoscopic to the finial preset temperature is summed to the duration calculated by the Planck formula.

In order to take into account the influence of the initial temperature of the product plate on the duration of refrigeration to the heat amount removed from the mass unit during freezing, a binomial multiplier was introduced ($1 + 0.0053\, t_i$).

Alyamovskii adjusted Ryutov's formula by assuming that the temperature distributed parabolically at the initial moment of freezing [2].

V.E. Kutsakova proposed a model for calculating the duration of refrigeration of an infinite plate, characterized by a simple mathematical formula and reduced to the Planck formula, taking into account the period of further freezing and the process of crystallization front movement. The model assumes a linear approximation of the temperature field of the material along the axis of temperature front distribution at the stages of refrigeration and the period of relaxation of the temperature field [3].

It is assumed that during refrigeration the heat transfer coefficient changes insignificantly in the course of the process. At the same time it is known that the linear and volume rates of refrigeration increase sharply as the heat transfer coefficient increases at a small thickness of the frozen layer, and, as the thickness increases, this effect is smoothed. However, under the conditions of adjustable heat sink, the heat transfer coefficient changes significantly in a two-zone nitrogen + air combined fast freezer, which should be taken into account during the process organization.

A new approach to solving the problem of determining the duration of refrigeration of biological objects was proposed by V.M. Stefanovskii, holding that, during cold treatment, it is important to know not the thickness of the frozen layer but the mass of unfrozen water by time point τ [4].

Thus, the considered analytical dependences for determining refrigeration duration have both an advantage in the relative simplicity of equations and general drawbacks, inherent in the Planck formula and its modifications: excessive simplifications in the problem statement and the representation of the temperature profile as a linear function.

When solving frost penetration problems according to Stefan, the phase boundary is represented as a line dividing the frozen and unfrozen zones of a product, which is usually called the crystallization front, which moves from the periphery to the center during refrigeration.

The statement of the Stefan problem for freezing a product plate in the interpretation of A.M. Brazhnikov considers a plate with a thickness of $\mathbf{2}\boldsymbol{l}$, initial temperature t_i, and cryoscopic temperature t_{cr} situated in an environment with temperature t_{en} ($t_{en} \leq t_{cr} \leq t_i$). The thermophysical characteristics of the plate are assumed to be constant within each zone and change in discrete steps at phase transformations. The refrigeration process consists of two stages. At the first stage, the product is cooled from its initial temperature to the cryoscopic temperature at the product surface, and, at the second stage, the crystallization front moves from the object surface to the object center. It is assumed that practically all moisture freezes up at the end of the second stage. The problem under consideration is nonlinear. It has no exact solution. Various approximate methods are used. One of them is the selection of a temperature function from experimental data, the search for a coefficient from boundary conditions, and the solution of a differential equation resulting from the Stefan condition [5].

Academician L.S. Leibenzon has developed a method that makes it possible to introduce values that characterize the boundary conditions on the object surface instead of derivates at the phase boundary [6].

The best results in determining the duration of the fast freezing of a large group of products were obtained by Russian scientists, in particular, by Venger, E.V. Semenov, I.E. Lobanov, B.S. Babakin, and M.I. Voronin, where the concerned problem of heat transfer in the conditions of continuous heat sink is based on an approximate method, the method of Leibenzon's integral relations, developed by A.M. Pirvedyan, V.A. Karpychev, and Brazhnikov [7, 8, 9].

Despite the abundance of theories and methods developed for solving this problem, in practice it is difficult to decide on a single model that would describe and take into account with a high accuracy all factors present in the problem statement. Therefore, various options and methods of freezing any biological object should take into account the geometric shape of a product, its structural heterogeneity, the specific changes of its thermophysical characteristics during the phase transition from moisture to ice, the availability of packaging, the quality of a product ready for processing, etc. The latest publications of Russian and foreign scientists, A.A. Tvorogova, P.B. Chizhova, V.O. Buyanov, and N.G. Craiver, are dedicated to the most important characteristic for frozen products, structure assessment by the condition of ice crystals depending on thermophysical and technological factors [10, 11, 12]. Buyanov has found out that intensive heat exchange, owing to high process velocities and low air temperatures, forms a finely crystalline ice structure in cheese mass, preserving at the same time the microstructure and consistency in a satisfactory condition. The consequence of fast freezing is almost full homogeneity of ice composition.

Changes in the consistency of cheeses during refrigeration and subsequent storage are associated with the loss of relatively free water. Moreover, changes in the consistency of frozen products can be avoided by adjusting the pH value.

Some of these factors are easy to take into account, while others are in a complex dependence on process stages. Therefore, it is rational to use the existing ready-made solutions to the theory of thermal conductivity during the analytical description of refrigeration processes; such solutions make it possible to reduce the problem of freezing products of complex geometric shapes to the refrigeration of products that are represented by equivalent bodies of simple forms, such as a plate, a cylinder, or a sphere [13].

Thus, the analysis of the kinetics of the fast refrigeration process in Buyanov's studies used mathematical methods of experimental design, and an object under study was represented as an equivalent body, i.e., an indefinite plate [11]. The knowledge of control mechanisms of these processes will make it possible to directionally regulate the composition and properties of frozen products and the degree of their intensity, preserving the initial nutritive and biological values. The effect of low temperatures on the microorganisms and general microflora of frozen cheese was studied by Russian scientists Buyanov, I.O. Larina, I.V. Buyanova, and O.V. Kriger [14].

At present, foreign specialists in creative collaboration with Russian scientists have developed a number of units to freeze products in liquid nitrogen:

from small units for 50–100 kg/h, made by Messer (the United States), to large continuous action units, made by Linde (the United States), Cryo-Quick by Air Products, Union Carbide by AGA, etc. [7, 15, 16]. The CER Chachak factory (Yugoslavia) produces cryogenic trizonal tunnel fast freezers that use liquid and gaseous carbon dioxide or liquid nitrogen. The specialists of CER Chachak note a number of technical and economic advantages of a machineless refrigeration system [7].

Foreign multizone cryogenic apparatuses have a capacity of 150–1500 kg/h; the tunnel length varies within 7–15 m; and the consumption of liquid nitrogen for the refrigeration of 1 kg of products is from 1.2 to 1.5 kg, the temperature of outgoing nitrogen vapors being from –50 to –70ºC.

The first study of dumpling refrigeration using liquid nitrogen was conducted in Russia by N.D. Abramov. Subsequent studies by N.A. Aleksandrova, G.D. Shabetnik, O.V. Anistratova, and B.N. Semenov showed the economic efficiency of liquid nitrogen for the refrigeration of endocrine–enzymatic special raw materials, as well as curds [7, 8, 9].

Li Ruixia, Wang Weicheng, and D. Coulomb studied the long-term development of refrigeration technology, including the modeling of cooling and frost-formation processes for cold accumulators [15, 16].

Of increasing interest lately has been a technology that includes two systems of air and cryogenic freezing and combines the advantages of both methods. The combined method uses combinations of flow-through systems with traditional machine systems.

Venger's works [7] describe the findings concerning heat exchange during the refrigeration of poultry carcasses by the combined method. The analysis has also shown that the nitrogen + air refrigeration reduces the process duration practically by 2–3 times compared to the air method and excludes product mass loss through shrinkage, because the instantly formed frozen layer hinders moisture evaporation from the product surface.

The comparative analysis of prospects for the development of refrigeration engineering and technology, conducted by Buyanov, showed the practicality of introducing multizone fast freezers with a combined cold supply system into the industry. Here he distinguished two main interrelated priorities: increasing the energy efficiency of freezers and their environmental safety [17]. He found out that high process velocities reduce the duration of refrigeration of batch dairy products and their mass.

Of interest is a freezer in which cold air is supplied to the initial refrigeration stage at a temperature of –50ºC, and then the vapors of boiling liquefied nitrogen at a temperature of –120ºC are supplied through the lower channels in the containers for final product freezing [18].

A two-module fast freezer efficiently solves the problems of fast refrigeration kinetics, the rational operating characteristics of the modules, the formation of a continuous technological process, and the resulting use of the cooling potential of the refrigerating environment [19].

Several tunnel freezers for the fast refrigeration of batch foods have been patented in the United States.

The purpose of this work is the development of a mathematical model for determining the duration of the fast refrigeration of foods in a two-zone modular fast freezer based on a nitrogen + air combined cold supply system with adjustable heat sink.

OBJECTS AND METHODS OF RESEARCH

For refrigeration modeling, a mathematical model developed by the Russian scientist Venger was taken as the basis [7]. In order to verify the adequacy of the developed mathematical model to the real refrigeration process, experiments were conducted at a special bench, designed at the Kemerovo Institute of Food Science and Technology, Russia (Fig. 1).

The experiments were conducted at the research laboratories of the departments of Heat and Cold Engineering, Technology of Milk and Dairy Products, and Technology of Fats, Biochemistry, and Microbiology at the Kemerovo Institute of Food Science and Technology, Russia.

The study of the specifics of the combined refrigeration method based on nitrogen and air systems of cold treatment was conducted at an experimental bench, the principal diagram of which is given in Fig. 1.

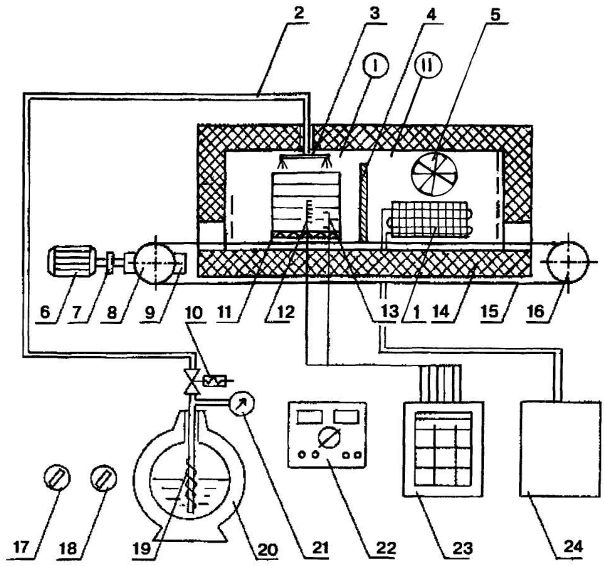

Fig 1. Principal diagram of the experimental bench: (1) Evaporators; (2) a liquid nitrogen pipeline; (3) a liquid nitrogen header with nozzles; (4) a partition; (5) a fan; (6) a direct-current motor; (7) a coupling; (8) a driving drum; (9) a reducer; (10) a solenoid valve; (11) trays; (12) a thermocouple unit; (13) heat meters; (14) a heat-insulated tunnel; (15) a cable; (16) a driven drum; (17) a controller of voltage fed to the heating element; (18) a controller of voltage fed to the fan; (19) a heating element; (20) a Dewar vessel, ADS-15; (21) an electric-contact manometer; (22) a voltage rectifier; (23) an interface unit; and (24) a refrigerating machine. I is the liquid nitrogen spraying (active effect) zone; II is the air cooling zone (temperature equalization by the product volume).

The main component of the bench is a heat-insulated tunnel (14), consisting of two zones.

The first zone (I) is a module of the nitrogen vapor effect (active zone). The second zone (II) is a module of the cold air flow effect. A sprayer (3) is installed in zone I. The nozzles are connected by a pipeline (2) to a system of cryogenic liquid supply, which consists of a Dewar vessel, ADS-15 (20), an electric-contact manometer (21), which is connected in series with a heating element (19) and which ensures an automatic, with the help of a solenoid valve (10), constant pressure within 0.02÷0.1 MPa in the Dewar vessel. The latter two are fed through a control, RKO-250-2A (17).

Nitrogen vapors enter zone II through windows in the upper part of heat-insulated partition 4. Evaporators I and fans 5 are installed in the partition, checkered one opposite another, creating a double-sided symmetrical air circulation. The number of the revolutions of the fans changes by the voltage control (18). Cold is supplied to zone II by refrigerating machine 24, which runs on Freon 404.

A system that transports the product to and through the tunnel consists of a carrier with meshed trays (11), which are connected by a cable (15) through the driving (8) and driven (16) drums, the bolt coupling (7), and the reducer (9) with the DC motor (6), the number of revolutions of which is regulated by the voltage rectifier (22).

The bench is equipped with two heat flow probes 13 and thermocouple unit 12, from which the signal is sent to interface unit 23, consisting of a microcontroller (which controls a measuring complex), an interface for data exchange with a PC, and an analog-to-digital circuit.

Chromel–copel thermocouples with a junction-point diameter of 0.3 mm were used as sensors for measuring the ambient temperature in the modules and in the sample under study during refrigeration. The rates of the nitrogen and air flows in the tunnel were measured by a thermal anemometer T-3 with a scale range of 0.1÷15 m/s.

During the experiments, measurement errors were observed and assessed by relative error values expressed in percent.

The freezer design makes it possible to maintain temperatures down to –120°C in zone I and down to –42°C in zone II and to create an air flow rate up to 8 m/s. During the experiments, the following parameters were measured and controlled:

– nitrogen and air vapor temperatures in product zones,
– the temperature field of the product under study,
– the density of the heat flow from the product to the cooling environment,
– the rate of air flow circulation in zone II, and
– the thickness of the portions of products under study.

The objects of refrigeration were packaged dairy products represented by the following collective groups: Dutch cheese portions and packaged curds of various fat contents. The product thicknesses were 0.025, 0.03, and 0.05 m, weighing from 0.1 to 0.5 kg.

The objects of research were placed on the bench transporter, and refrigeration was conducted from the initial temperature of 15°C to the preset volume-averaged product temperature of –20°C, which was equal to the subsequent storage temperature. The junction point of the thermocouples was introduced into a sample of a given experimental series all through its thickness at equal lengths. The thermocouple readings were recorded by the microcontroller every 4 s.

The refrigeration experiments were conducted under the following conditions: fast refrigeration at the temperature of nitrogen vapors in zone I from –50 to –90°C and in zone II at air temperatures from –20 to –40°C and an air circulation rate of 5 m/s. In zone I of the freezer, nitrogen vapors actively affected the product surface, and in zone II the process heat exchange continued between the product and the air flow but less intensively until the preset final temperature was reached.

The experimental studies of heat exchange processes during refrigeration were conducted using mathematical methods of experiment design. The main experimental material in the heat exchange studies was thermograms of the combined refrigeration process and the averaged-integral values of the heat flow density, which served as the basis for determining the duration and average rate of refrigeration.

The average rate of refrigeration was calculated in line with the recommendations of the International Academy of Refrigeration as a ratio of the distance from the product surface to its thermal center to the time lag between a temperature of 0°C reached on the surface and that in the thermal center 10°C lower than the cryoscopic temperature.

The density of the heat flow was measured with heat meters DPTP. The true value of the heat flow density was derived from an expression:

$$q = A \cdot K \cdot K_t,\ \mathrm{W/m^2}, \quad (1)$$

where A is the value of a signal from the heat meter, mV; K is the operating coefficient of the heat meter, $W/(m^2 \cdot mV)$;
$K_1 = 165.26\ W/(m^2 \cdot mV)$, $K_2 = 160.73\ W/(m^2 \cdot mV)$;
K_t is a dimensionless temperature correction coefficient, which takes into account the signal error of the heat meter at low temperatures:

$$K_t = 0.000017t^2 + 1.005,$$

where t is the heat meter's temperature at a certain time point,°C.

RESULTS AND DISCUSSION

The situation of heat exchange during fast refrigeration under consideration is represented as a system of modules, each of which can independently provide the necessary heat sink conditions for the process. In addition, the specifics of a multizone combined system of cold supply are taken into account; i.e.:

– in zone (module) I, the nitrogen vapors freeze the product to a volume-averaged temperature, which is equal to the product's cryoscopic temperature;
– in zone II (which may consist of several modules) the product is affected by cold air, cooled by nitrogen vapors escaping from module I (partially), and by the

(main) machine refrigeration system to the preset final temperature of the product.

The model under development was based on the method of Leibenzon's integral relations with account for the following assumptions:
– the product has the shape of an indefinite plate;
– the heat exchange conditions are symmetrical;
– the temperature of the cooling environment is constant within each stage; and
– the thermophysical characteristics of the product change in discrete steps during the water phase transfer, and they are constant within one phase of the water condition.

For the analytical description of the fast refrigeration process, conventional simplifications were used, and the problem was solved by dividing the whole process into three stages, which were considered sequentially:
– the first stage is refrigeration to the cryoscopic temperature on the product surface;
– the second stage is refrigeration to the cryoscopic temperature in the product's thermal center; and
– the third stage is refrigeration of the frozen product to the preset temperature in the thermal center.

First stage. The known solution is based on the hypothesis of the presence of the "temperature front," which proliferates from the surface to the central layers of the object with the final velocity.

The accurate problem statement consists in solving a thermal conductivity equation:

$$\frac{\partial t}{\partial \tau} = a \cdot \frac{\partial^2 t}{\partial x^2}, \quad (2)$$

that satisfies the initial:

$$t(x,0) = t_i = \text{const}, \quad (3)$$

and boundary conditions:

$$\left(\frac{\partial t}{\partial x}\right)_{x=r} = 0; \quad \left[\frac{\partial t}{\partial x} - \frac{\alpha}{\lambda}\cdot\left(t - t_{av}\right)\right]_{x=0} = 0, \quad (4)$$

where $t_{av} < t_{cr} < t_i$, t_{av} is the ambient temperature, °C; t_i is the initial product temperature, °C; t_{cr} is the product cryoscopic temperature, °C;
$\delta = 2r$ is the thickness of the product plate, m;
a is the thermal conductivity coefficient, m^2/s;
and λ is the thermal conductivity coefficient of the product, W/(m·K).

In order to solve the problem under consideration, dimensionless variables were introduced.

$$\theta = \frac{t_i - t}{t_i - t_{av}}; \quad \xi = \frac{x}{r}; \quad Fo = \frac{a \cdot \tau}{r^2}; \quad Bi_2 = \frac{\alpha}{\lambda_2} \cdot r, \quad (5)$$

and moreover:

$$\theta_{cr} = \frac{t_i - t_{cr}}{t_i - t_{av}}. \quad (6)$$

The duration of the first phase of the refrigeration stage:

$$Fo_I^a = \frac{1}{12 Bi_2} \cdot \left[Bi_2^2 + 4Bi_2 - 8\ln\cdot\left(1 + 0{,}5Bi_2\right)\right]. \quad (7)$$

The duration of the second phase of the refrigeration stage:

$$Fo_I^b = \frac{Bi_2 + 3}{3Bi_2} \cdot \ln \frac{2}{\left(Bi_2 + 2\right)\cdot\left(1 - \theta_{cr}\right)}. \quad (8)$$

The full duration of the first stag refrigeration to the cryoscopic temperature on the plate surface, is determined as follows:

$$Fo_I = Fo_I^a + Fo_I^b. \quad (9)$$

The calculation diagram of the product at the refrigeration stage is given in Fig. 2.

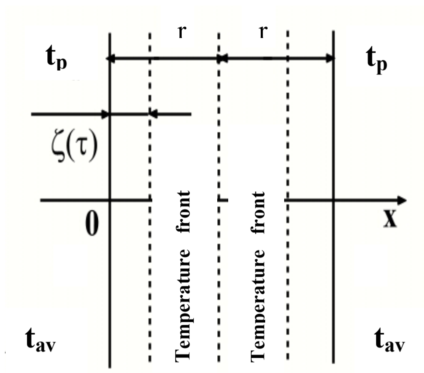

Fig. 2. Formation of the temperature front at the refrigeration stage.

Second stage. The plate is split into two zones: frozen and unfrozen. It is assumed that the frozen zone is $0 \le x \le \tilde{x}(\tau)$, and the unfrozen zone is $\tilde{x}(\tau) \le x \le r$, where $x = \tilde{x}(\tau)$ is the boundary of the crystallization front (Fig. 3).

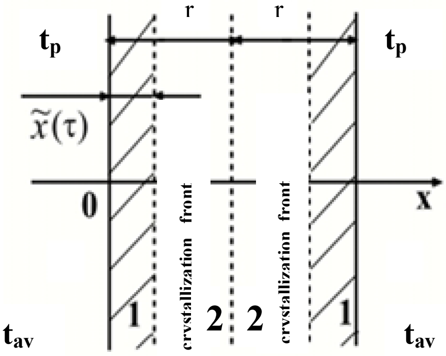

Fig. 3. The calculation diagram of the product at the refrigeration stage.

The solution of the above equations should be subject to the following conditions:

$$t(x,0)=t_{cr}+\frac{\alpha}{2\cdot r\cdot\lambda_2}\cdot\left(t_{av}-t_{cr}\right)\cdot\left(x^2-2\cdot r\cdot x\right), \quad (10)$$

$$\left[\frac{\partial t_1}{\partial t}-\frac{\alpha}{\lambda}\cdot(t_1-t_{av})\right]_{x=0};\quad \left(\frac{\partial t}{\partial x}\right)_{x=r}=0, \quad (11)$$

$$\left(t_2-t_1\right)_{X=\tilde{X}}=0;\quad (t)_{X=\tilde{X}}=t_{cr}. \quad (12)$$

In addition, functions t_1 and t_2 must satisfy the Stefan condition:

$$\left(\lambda_1\cdot\frac{\partial t_1}{\partial x}-\lambda_2\cdot\frac{\partial t}{\partial x}\right)_{X=\tilde{X}}=L\cdot W\cdot\omega\cdot\rho\cdot\frac{d\tilde{x}}{d\tau}, \quad (13)$$

where λ_1 is the coefficient of thermal conductivity of the frozen zone,
W/(m·K);
L is the heat of the phase transfer of crystallization, J/kg;
W is the product's relative moisture content, %; and
ω is the share of frozen-out moisture, unit.

Then the problem at hand is reduced to a dimensionless form:

$$\theta_1=\frac{t_i-t_1}{t_i-t_{av}};\ \xi=\frac{x}{r};\ \xi_{(\tau)}=\frac{\tilde{x}(\tau)}{r};$$

$$\theta_{cr}=\frac{t_i-t_{cr}}{t_i-t_{av}};\ Fo=\frac{a\cdot\tau}{r^2};\ Bi_1=\frac{\alpha}{\lambda_1}\cdot r. \quad (14)$$

The formula for determining the duration of the second stage has the following form:

$$Fo_{II}=\frac{\beta}{2\eta}\cdot\frac{Bi_1+2}{Bi_1\left(1-\theta_{cr}\right)}+\frac{1}{3}\cdot\left[\left(\frac{Bi_1+1}{Bi_1}\right)^2\cdot\ln\left(Bi_1+1\right)-\frac{2Bi_1+1}{Bi_1}\right], \quad (15)$$

where $\beta=\dfrac{L\cdot\omega\cdot\rho\cdot a_2\cdot W}{\lambda_2\cdot\left(t_i-t_{av}\right)};\quad \eta=\dfrac{\lambda_1}{\lambda_2}.$

The total duration of the product stay in zone (module) I is calculated by summarizing Fo_I+Fo_{II}.

As was noted above, in terms of the production process, by the time the product transfers from zone I to zone II of the multizone combined fast freezer, the volume-averaged temperature of the product must be equal to the cryoscopic temperature. In order to determine the time of transfer, it is necessary to calculate the volume-averaged temperature of the product at this refrigeration stage. To this end, let us use Venger's methodology, according to which the product plate in this situation is split into three zones: frozen zone I with a thickness of ε_1, unfrozen zone II, and zone III, frozen from the other side of the product, with a thickness of ε_2 [7].

Since various thicknesses of the frozen layers are assumed, the intensities of heat exchange also differ. Therefore, an asymmetry coefficient k is introduced.

Then, the formula for determining the volume-averaged dimensionless temperature is as follows:

$$\theta_v=\frac{Bi_2}{4}\left(\frac{\varepsilon_1^2}{Bi_2\varepsilon_1+1}+\frac{\varepsilon_2^2}{Bi_2\varepsilon_2+k}\right)+\frac{\theta}{6}\left(2-\varepsilon_1-\varepsilon_2\right)^2. \quad (16)$$

If the symmetrical conditions of heat sink are ensured, then $\varepsilon_1=\varepsilon_2=1$ and $k=1$. Then:

$$\theta_v=\frac{Bi_2}{4}\left(\frac{\varepsilon^2}{Bi_2+1}\right). \quad (17)$$

After the transfer to zone (module) II of the multizone freezer, the refrigeration stage still continues but under different heat-sink conditions, i.e., under the different values of criterion Bi. Here it is important to note that, while in zone I the product is cooled by nitrogen vapors at temperature t_{av1} (by the data of experiment t_{av1} = –70°C), in zone II it is cooled by the air flow at temperature t_{av2} (by the experimental data, t_{av2} = –30°C). In addition, the volume-averaged temperature of the product in zone II continues to decrease until the crystallization fronts meet, which will finally mean the completion of the refrigeration stage, after which the further freezing stage begins until the preset temperature is reached: either the volume-averaged temperature or that in the thermal center of the product, depending on the process conditions.

In order to determine the span of refrigeration completion in zone II, we have to determine the temperature in the thermal center of the product, θ_c, at the transition time, as well as the thickness of the frozen layer, ε. Taking into account the symmetry of the problem under consideration, apparently, $\theta_c=\theta/2$. Then the sought time interval is

$$Fo=\frac{\Delta\varepsilon}{\beta\theta_{cr}\theta_c}-\frac{\eta}{2\beta\theta_{cr}\theta_c^2}\ell n\left(\frac{2\theta_c Bi\varepsilon+2\theta_c-\eta Bi}{2\theta_c Bi\Delta\varepsilon+2\theta_c-\eta Bi}\right). \quad (18)$$

Third stage. Here the refrigeration of the frozen product to the preset final volume-averaged temperature of the body is considered (Fig. 4).

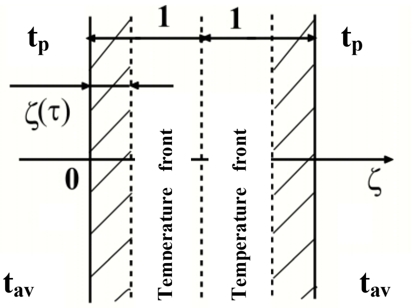

Fig. 4. Calculation diagram of the product at the further freezing stage.

The following equation is solved:

$$\frac{\partial T_1}{\partial Fo} = \frac{\partial^2 T_1}{\partial \xi^2}, \qquad (19)$$

at the following boundary conditions:

$$\left(\frac{\partial T_1}{\partial \xi}\right)_{\xi=1} = 0; \quad \left(\frac{\partial T_1}{\partial \xi} - Bi_1 \cdot T_1\right)_{\xi=0} = -Bi_1. \qquad (20)$$

The result of the solution is a formula for determining the duration of the plate's further freezing down to volume-averaged temperature t_v:

$$Fo_{III} = \frac{Bi_1 + 3}{3Bi_1} \cdot \ln \frac{2(1-\theta_{kp}) \cdot (Bi_1 + 3)}{3(Bi_1 + 2) \cdot (1 - \theta_{1v})}. \qquad (21)$$

The total duration of the refrigeration of the product plate is determined as the sum of the durations of individual stages:

$$Fo = Fo_I + Fo_{II} + Fo_{III}. \qquad (22)$$

Formula 15 includes the value of a share of frozen-out moisture, which is associated with the volume-averaged temperature of the product plate when the center reaches the cryoscopic temperature. In order to determine the share of frozen-out moisture of real objects of research, Ryutov's formula has been used, which takes into account the relative amount of nonfreezing bound moisture (σ), as a share of the amount of dry substances:

$$\omega = \left(1 - \sigma \frac{1-\omega_p}{W}\right)\left(1 - \frac{t_{kp}}{t_v}\right), \qquad (23)$$

where W is a mass share of moisture, unit share;
ω_p is the mass share of freezing-out moisture, unit share.

Table 1 presents the results of calculating the share of frozen-out moisture in the objects of research in a wide temperature range.

Table 1. Calculation results of the share of frozen-out moisture

Refrigeration temperature, minus °C	Curds of 5% fat content, W = 74.5%	Dutch cheese bar, W = 40.5%
-10	0.699	0.600
-15	0.799	0.730
-20	0.849	0.800
-25	0.879	0.840
-30	0.899	0.870
-35	0.913	0.880
-40	0.924	0.882
-45	0.932	0.888
-50	0.939	0.890
-55	0.945	0.893
-60	0.949	0.900
-70	0.956	0.920

When studying the thermophysical processes of refrigeration technology, the volume-averaged temperature is viewed as a very important value. In nonstationary processes, characteristic of fast freezing, it changes in time because the temperature field of a body changes, as well as its thermophysical characteristics. Therefore, the problem of determining this temperature becomes more complicated.

Simplifying the problem and assuming that the thermophysical characteristics within one phase are constant, we may represent the dimensionless volume-averaged temperature of the product by an integral:

$$\theta_{v1} = \int_0^1 \theta_1(\xi, Fo_{II})d\xi. \qquad (24)$$

From the integral relation we have:

$$\theta_1(\xi, Fo_{II}) = \chi_1(Fo)\xi + \chi_2(Fo), \qquad (25)$$

where $\chi_1 = -\frac{Bi_1(1-\theta_k)}{1+Bi_1\xi}$; $\chi_2 = \frac{\theta_1 + Bi_1\xi}{1+Bi_1\xi}$. (26)

At time point Fo_{II}, considered in the problem, ξ = 1, then:

$$\chi_1 = -\frac{Bi_1(1-\theta_k)}{1+Bi_1}; \qquad \chi_2 = \frac{\theta_1 + Bi_1}{1+Bi_1}. \qquad (27)$$

Inserting expression 27 into 25, we obtain:

$$\theta_1(\xi, Fo_{II}) = -\frac{Bi_1(1-\theta_k)}{1+Bi_1}\xi + \frac{\theta_1 + Bi_1}{1+Bi_1}, \qquad (28)$$

We introduce expression 28 into integral 24 and find:

$$\theta_{v1} = \frac{\theta_k(2+Bi_1) + Bi_1}{2(1+Bi_1)}. \qquad (29)$$

Taking into account $\theta_k = \frac{t_{cr} - t_k}{t_{cr} - t_{av}}$, we obtain:

$$t_v = t_i - \frac{\theta_k(2+Bi_1) + Bi_1}{2(1+Bi_1)}(t_i - t_{av}). \qquad (30)$$

Then the final temperature in the product center that corresponds to the preset volume-averaged temperature will be equal to:

$$t_k = \frac{2 \cdot t_v \cdot (1+Bi_1) - Bi_1 \cdot t_{av}}{2 + Bi_1}. \qquad (31)$$

Analysis of the findings shows that, at the fast refrigeration of the research objects in nitrogen vapors in zone I of the combined fast freezer, the temperature of the product surface reaches very quickly the cryoscopic value. As a result, the duration of the first stage of the process tends to zero. In this case, the heat front hypothesis, which underlies the model, is inapplicable because the product surface reaches the cryoscopic temperature much earlier than the temperature front reaches the thermal center. Analytically, this leads to the fact that the second summand $Fo_I^{б}$ (15) becomes negative, which contradicts the criterion of formula applicability for calculating Fo. Consequently, if this criterion is inapplicable, the mathematical formula for calculating the dimensionless time of the whole refrigeration process will have the following form:

$$Fo = Fo_{II} + Fo_{III}\ . \tag{32}$$

Tables 2 and 3 show the thermophysical characteristics of dairy products used as the research objects and for calculations.

Table 2. Thermophysical characteristics of the research objects

Product name	Mass share of moisture, %	Density, kg/m^3	Thermal capacity, kJ/(kg·K)	Thermal conductivity, W/(m·K)	Temperature conductivity, 10^7 m/s^2
Dutch cheese, F_{ds}= 45%	40.5 ± 0.2	1070	2.5	0.35	1.31
Curds, 5% fat content	74.5 ± 0.2	962	3.27	0.43	1.37

Table 3. Thermophysical characteristics of the frozen products [20]

Product name	Cryoscopic temperature, °C	Density, kg/m^3	Thermal capacity kJ/(kg·K)	Thermal conductivity, W/(m·K)	Temperature conductivity, 10^7 m/s^2
Dutch cheese, F_{ds} = 45%	Minus 6.2	1025	1.28	1.10	8.38
Curds, 5% fat content	Minus 3.0	960	2.18	1.15	5.49

The predicted results for the duration of refrigeration compared to the experimental data given in Buyanov's works [19] show that the maximum value of the maximum relative error does not exceed 14%, which indicates the adequacy of the newly developed mathematical model to the real refrigeration process. Thus, a mathematical model has been developed for determining the duration of the fast refrigeration of foods in a two-zone modular fast freezer based on a nitrogen + air combined cold supply system with adjustable heat sink.

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